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TRANSITION FROM ORDER TO CHAOS IN THE WAKE OF A RECTANGULAR CYLINDER

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Abstract. *Although the transition from order to chaos in nonlinear systems has been verified and detailed for a number of experiments on closed flow, the literature on this subject is scarce or non-existent for external flows. In this work, we have studied the transition to chaos in the onset of the wake of the flow past a rectangular cylinder, a typical external flow. The experimental data have been obtained with a hot-film anemometer in a hydrodynamical tunnel. We found that by increasing the Reynolds number, the corresponding periodic flow has become unstable by an intermittence scenario, a period-doubling cascade, or a Hopf bifurcation. The degree of chaotic behavior has been verified by applying concepts from nonlinear dynamics to the experimental data, therefore relating the underlying physics of the system to the elements of mode interaction and chaos.*

Keywords: *Experimental chaotic time series, transition, wake of a rectangular cylinder*

1. INTRODUCTION

Understanding of flow around rectangular cylinders is of special interest in aerodynamics, so that since very earlier attempts have been made in the literature to explain the transition (route) from laminar to turbulent flow (Fiedler-Ferrara, 1994).

It has been observed that for some conditions the transition to chaotic states and to turbulence on closed flows occurs according to scenarios in general associated with low-dimensional dynamical systems. Although this association has been verified and detailed for a number of experiments on closed flow, the literature on this subject is scarce or non-existent for external flows. The question is whether the same association is possible for external flows. In this work, we have studied experimentally the transition to chaos in the onset of the wake of the flow past a rectangular cylinder. Additionally, the degree of chaotic behavior has been verified by applying concepts from nonlinear dynamics to the experimental data.

2. EXPERIMENTAL FACILITY AND METHODS

2.1. Water Tunnel

The experimental tests have been conducted in an open-circuit vertical water tunnel driven by gravitational action, sketched in Fig. 1. Its test section, spanning 146 x 146 x 500 mm, has four wide Plexiglas windows that make it specially suitable for flow visualization tests.

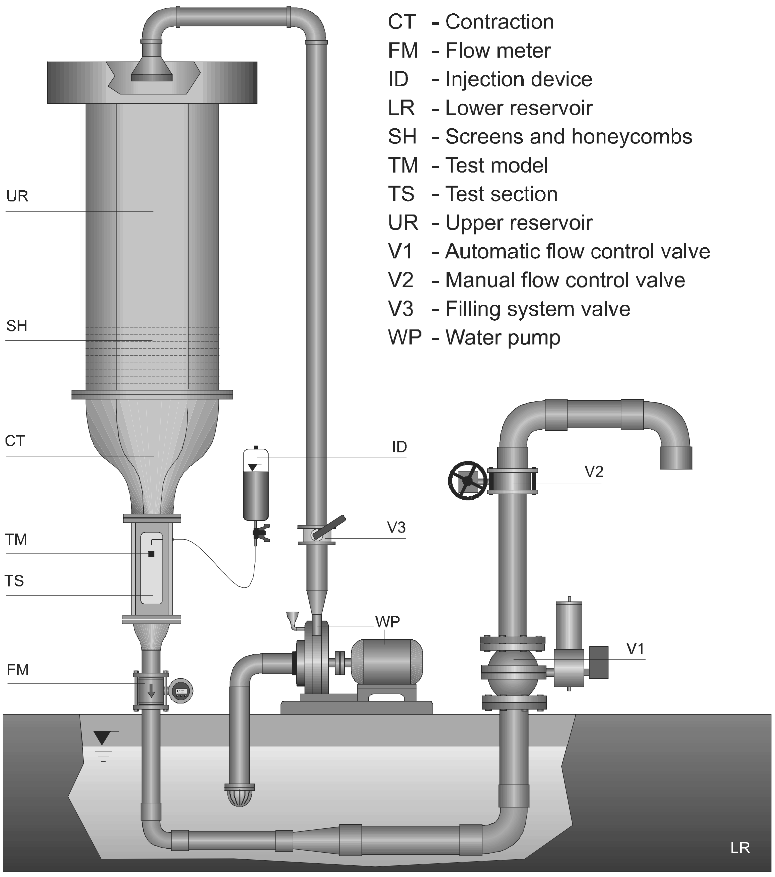

Figure 1 – Schematic illustration of the vertical water tunnel.

The water tunnel may be employed in two modes of operation: continuous and blow-down. In the first mode, with the water pump in operation, the flow control valves (V1 and V2) and the filling system valve (V3) are adjusted in order to keep constant the water level inside the upper reservoir. This mode allows the conduction of tests with long duration, constant free stream velocity, and turbulent level of less than 1%. In the second mode of operation, after the upper reservoir has been filled up, its left to rest for at least 15 minutes. Then, the flow control valves are partially opened, and the test takes place as the water level inside the upper reservoir continuously decreases.

2.2. Test Model

During the tests, two rectangular cylinders spanning $W \times H \times L$ have been employed, with main dimensional characteristics summarized on Table 1. The models have been machined, grinded and polished in order to achieve good dimensional quality, sharp edges and smooth surfaces. The test models have been firmly attached to the test section rear window with null incidence angle, that is, with the smaller edge (B) facing upstream.

Table 1 - Rectangular cylinders main characteristics.

Model	Material	Width W (mm)	Height H (mm)	Length L (mm)
1	Stainless steel	3,01	5,99	146,5
2	Aluminum	6,13	12,37	146,5

2.3. Experimental Technique

For the instantaneous velocity measurement, it has been employed a constant temperature anemometer Dantec® StreamLine model 90N10, employing a CTA 90C10 module and a fiber probe model 55R11, connected to a computer-based data acquisition system. The anemometer is controlled through the application software StreamWare version 1.11, supplied by the manufacturer.

The probe, installed on an L-shaped support model 55H22, has been introduced in the test section through one of its lateral windows and placed in the cylinder wake. The probe support has been attached to a positioning device, and the probe distance from the test model have been settled to $5H$ downstream and approximately $2H$ sidewise.

The velocity signal, without passing through any filters, has been dicretised and recorded using a sampling rate ranging from 1kHz to 2kHz. A typical record of sampled data had 2×10^3 data points.

2.4. Dimensionless Parameters

A dimensionless number that exerts a major influence on the flow around cylindrical bodies is the Reynolds number, here defined as

$$Re = \frac{VW}{\nu} \tag{1}$$

where V is the free stream velocity, W is the cylinder characteristic diameter (in the present work, the cross section width) and ν is the fluid cinematic viscosity. The free stream velocity has been measured with an electromagnetic flow meter (FM in Fig. 1). The water cinematic viscosity has been calculated from an empiric expression, given as a function of temperature. The Reynolds number uncertainty has been estimated to be within 5%.

The Strouhal number is defined as

$$Sr = \frac{fW}{V} \tag{2}$$

where f is the vortex shedding frequency, computed applying the discrete fast Fourier transform on each recorded run of velocity signal. The vortex shedding frequency has been taken as the one for which the spectral power density attained a local maximum value. When it has been found two peaks in the spectra, the frequency associated to the most important one has been called “dominant”, while the other, “secondary”.

The cylinder side ratio, given by the relation between the height and width of its rectangular cross section, has been 2 for both test models. The cylinder aspect ratio, defined as the relation between its length L and characteristic diameter W, has been found to be 49 and

24 for the test model number 1 and 2, respectively. The geometric blockage ratio, given by the relation between the cylinder frontal area and the test section transversal area, has been kept under 4,9%, so that no correction procedure has been employed.

3. NOISE REDUCTION

To perform noise reduction, we started embedding a single time series in a state space. Following the approach established by Takens (1991)

$$\mathbf{x}_i(t) = \{x_i(t), x_i(t+\tau), \cdots, x_i(t+[m-1]\tau)\} \quad (3)$$

where x_i is the measured velocity, τ is a adequate time delay and *m* is the embedding dimension, Eq. (3) provides a continuous embedding space for the system. Although the reconstructed attractor is not identical to original, it preserves its geometric and dynamical characteristics.

In this work, we have used a time delay of the order of the autocorrelation time of the time series (Fiedler-Ferrara, 1994). The choice of the embedding dimension *m* and noise reduction has been performed by using the singular value decomposition of the covariance matrix. As in Broomhead (1986) and in Cambraia (1997) a minimum relative value for the eigenvalues of the covariance matrix has been established for choosing the embedding dimension and for eliminating white and Gaussian noises.

4. RESULTS

Experimental evidence shows that the wake of a rectangular cylinder becomes unstable with appearing of the characteristic Kármán vortex at a Reynolds number about 50. This corresponds to a point attractor in phase space that loses its stability and, through a Hopf bifurcation becomes a limit cycle.

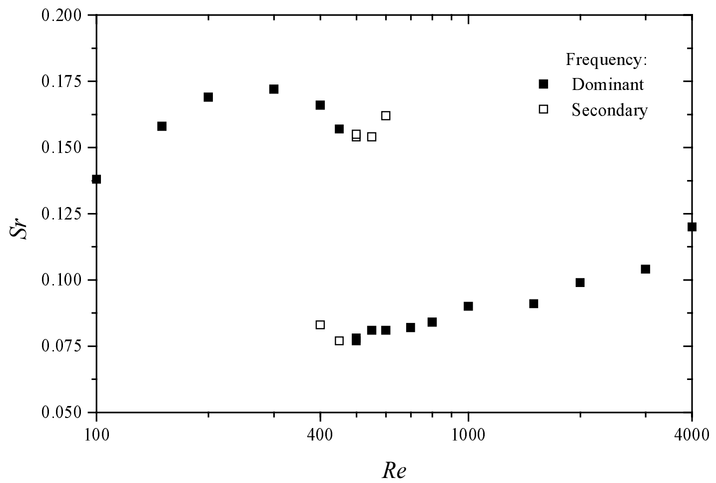

Figure 2 - Strouhal × Reynolds relationship for a rectangular cylinder ($H/W = 2$)

Figure 2 presents the Strouhal curve for a rectangular cylinder with a side ratio $H/W = 2$. As one can see, for Reynolds number up to 400, the Strouhal curve has a smooth behavior. At Reynolds number 400 a lower frequency appears in the flow. This frequency is a subharmonic of the higher, and it has occurred as a result of a period doubling bifurcation. In that discontinuity region, as pointed out by Okajima (1982) and Lindquist *et al*. (1998), it occurs a change in the vortex shedding dynamics, evident from the frequency analysis shown in the Fig. 3.

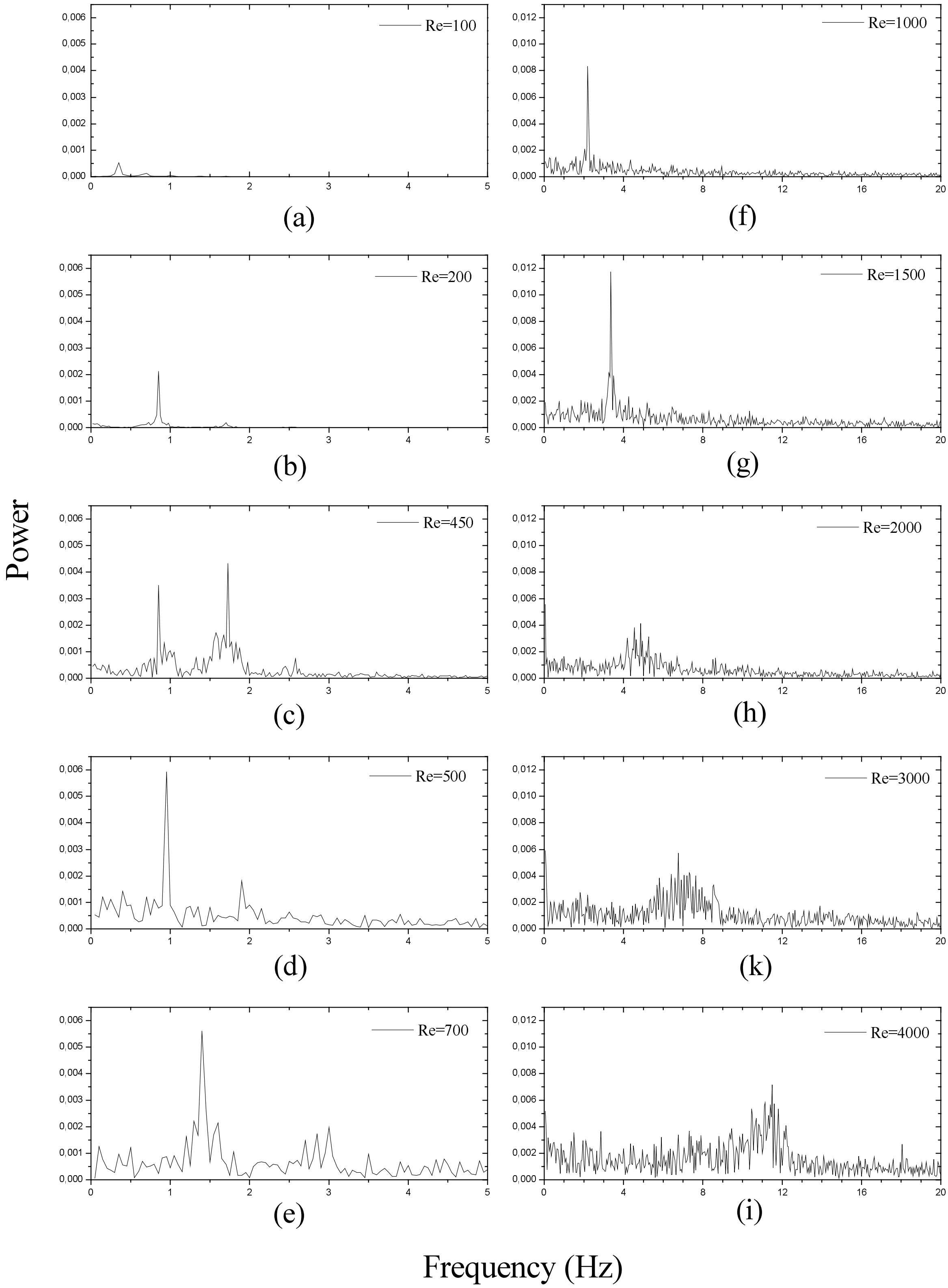

Figure 3: Density power spectra for Reynolds numbers ranging from 100 to 4000.

With increasing Reynolds number, in the range 400<*Re*<600, the power of the lower-frequency component increases becoming dominant, while the power of the higher one decreases up to the disappearance of this component (Fig. 3(f)).

At higher Reynolds number, the secondary frequency disappears and the power spectra becomes progressively broader, corresponding seemly to a chaotic attractor in phase space, although it can be possible to identify a perseverant characteristic frequency (Fig.3(i)).

After noise reduction performed by SVD, the attractors were reconstructed in a embedding dimension m=3 (Fig. 6), m=7 (Fig. 7) and m=10 (Fig. 8) employing the time delay $\tau = 3,5.10^{-1}$s (Fig. 6), $\tau = 4.10^{-2}$s (Fig. 7) and $\tau = 10^{-2}$s (Fig. 8). Figures 6,7,8(a) present the projection of this attractor onto the phase coordinates U(t) and U(t+τ) and Figures 6,7,8(b) show the intersection of trajectories with a plane at 45° from the horizontal constituting the Poincaré section.

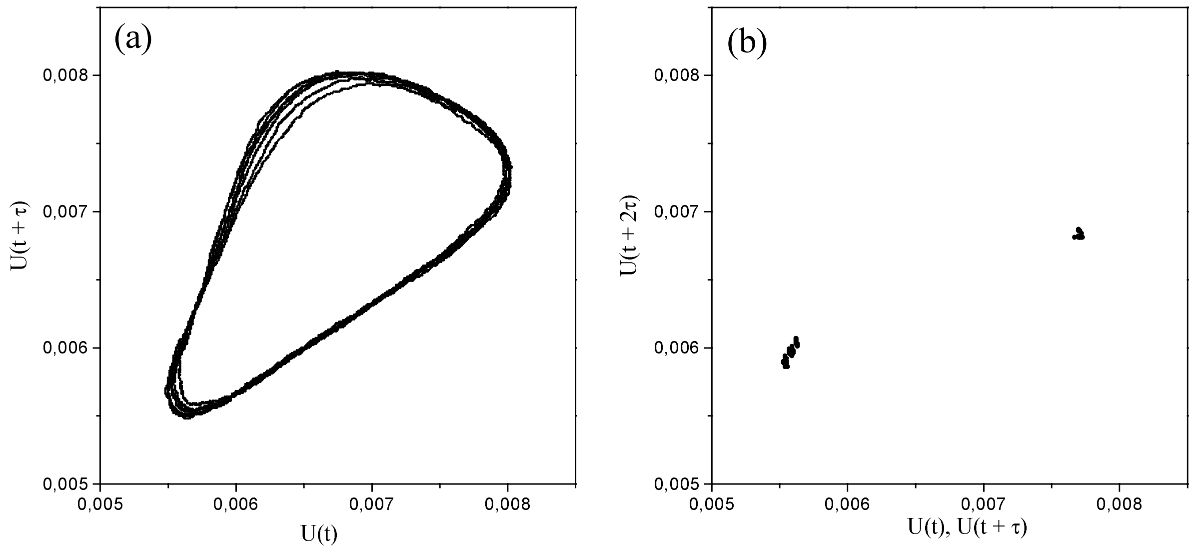

Figure 6: Projection of the reconstructed attractor in a 3-dimensional space (a) and the Poincaré section at 45° from the horizontal, for *Re*=100.

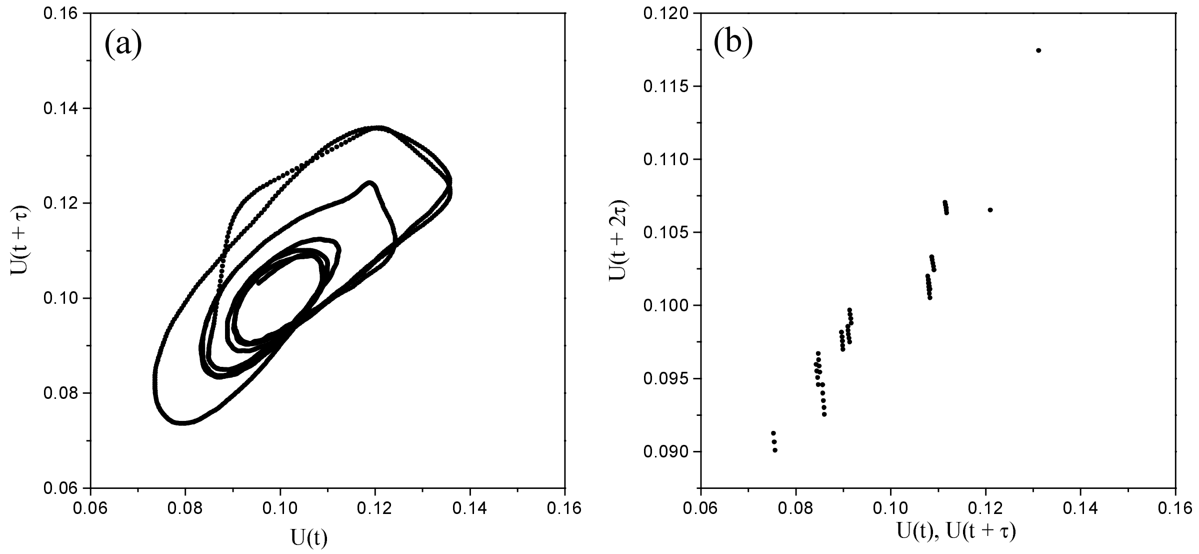

Figure 7: Projection of the reconstructed attractor in a 7-dimensional space (a) and the Poincaré section at 45° from the horizontal, for *Re*=450.

Figure 6 corresponds to the periodic case, while Fig. 7 presents the data after period doubling bifurcation and Fig. 8 shows a possible chaotic data, being Fig. 8(b) similar to that presented by Williams-Stuber (1990) for the chaotic case.

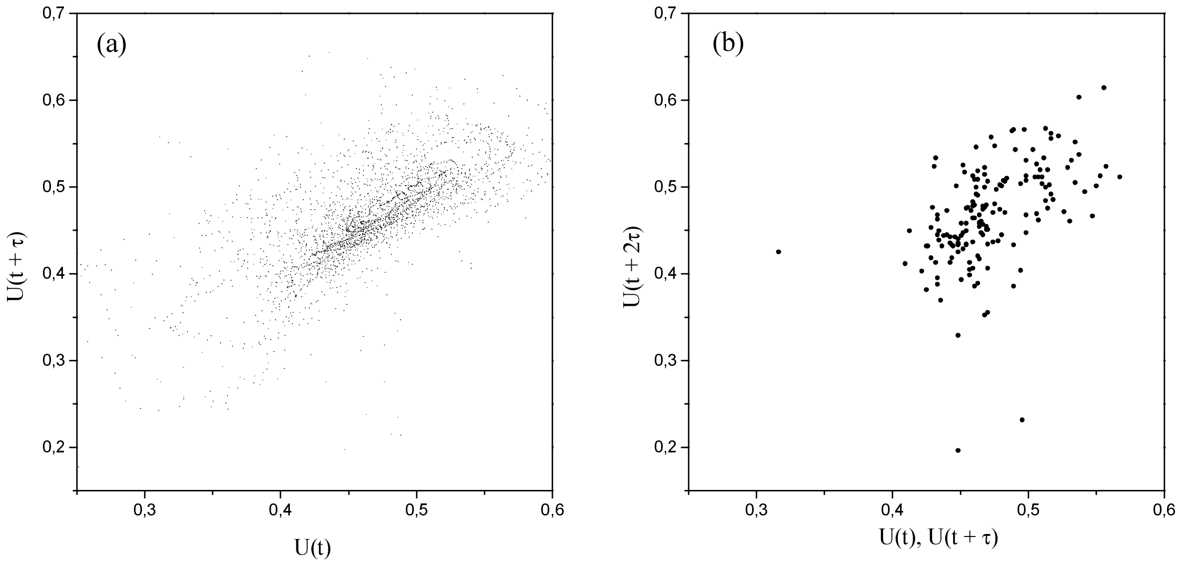

Figure 8: Projection of the reconstructed attractor in a 10-dimensional space (a) and the Poincaré section at 45° from the horizontal, for *Re*=4000.

5. CONCLUDING REMARKS

In this work, it has been confirmed that the route from order to chaos in the wake of a rectangular cylinder starts with a Hopf bifurcation and it has been found that it is followed by a period doubling bifurcation and by a re-stabilization of the periodic behavior. It still is left up to further investigation the behavior for remaining range of the Reynolds number, whereas from the experiment alone we could not verify whether the corresponding periodic flow has became unstable by an intermittence scenario, a period-doubling cascade, or a Hopf bifurcation, although the signal behavior at Reynolds number greater than 2000 seems to be chaotic. In future works, we will study with detail the phenomenon behavior in Reynolds number range 1500 to 4000.

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